Frequently Asked Questions

Approaches to Instruction and Assessment

"The primary purpose of assessment and evaluation is to improve student learning."
Ontario Curriculum - Mathematics, Grades 1-8, and Growing Success

"When we make thinking visible, we get not only a window into what students understand, but also how they are understanding it."
Ritchhart, Church & Morrison, 2011

1. What instructional approaches best support a comprehensive mathematics program?

In designing a comprehensive and balanced mathematics program, it is important to consider a problem-based learning environment which allows the teacher to focus on the development of conceptual and procedural understanding, skill development, and problem-solving. This environment fosters a culture of inquiry and uses engaging and intriguing rich tasks to support a comprehensive mathematics program.

The Ontario Mathematics Curriculum, grades 1-8 (2005) states "problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction."

Within the context of a balanced mathematics program, the three part lesson framework is a mindset which allows for the thinking and doing of mathematics and provides opportunities for students to consolidate and reflect on their learning. The use of rich tasks within this framework is crucial, although their use alone is not sufficient. The learning environment and the implementation of the task, including listening and responding to students through effective questioning, are what make the task rich. Piggott (2007) states, "much of what makes a rich task 'rich' is the environment in which it is presented, which includes the support and questioning that is used by the teacher and the roles that learners are encouraged to adopt." To learn more about student inquiry, click here.

"Asking clarifying and extending questions that prompt clarification, that gather and generate different ideas and approaches and challenge the validity of ideas discussed prompt all students to think about and make connections between their mathematical thinking and the class mathematical discussion."
Ministry of Education, 2010

A balance must be found between independent and collaborative work, and guided and direct instruction through targeted, purposeful and intentional planning based on student learner profiles. *Learning for All* (2013) provides information on the learner profile in chapter 4. Students should have the opportunity to work in a variety of collaborative groupings, and be given multiple opportunities to discuss and demonstrate their math understanding. Bruce (2007) states, "math is…a social endeavor…where thinking, talking, agreeing, and disagreeing are encouraged." Groups may change, as Krpan (2013) states, "based on the topics explored and the students' individual needs."

> *"The role of the teacher during whole-class discussion is to develop and build on the personal and collective sense-making of students rather than to simply sanction particular approaches as being correct or demonstrate procedures for solving predictable tasks."*
>
> Stein, Engle, Smith & Hughes, 2008

2. What assessment approaches best support a comprehensive mathematics program?

When we assess our students within a comprehensive mathematics program, we are gathering information about their learning that informs our teaching. Assessment information should be triangulated in the form of observations, conversations and products. The Ministry of Education (2010) states, "we obtain assessment information through a variety of means, which may include formal and informal observations, discussions, learning conversations, questioning, conferences, homework, tasks done in groups, demonstrations, projects, portfolios, developmental continua, performances, peer and self-assessments, self-reflections, essays, and tests…Teachers will ensure that students' demonstration of their achievement is assessed in a balanced manner with respect to the four categories of the achievement chart."

Observations can be a focused opportunity for observing student thinking and understanding in the math classroom. We might observe, for example, that a student has trouble selecting a problem-solving strategy when dealing with a proportional reasoning problem, and plan a targeted and precise guided lesson to move their mathematical thinking along. Or, we might notice that a student is struggling with multiplication, because they are still developing their basic fact fluency. Davies (2007) states, "some learning can only be observed…the record of observations becomes evidence."

Student-teacher conversations are another opportunity to gather very specific assessment evidence about student learning. Through mathematical discourse, students show how they understand the mathematics by explaining their thinking, discussing their strategies, and illuminating any struggles they are having. Conversations allow us to give very targeted descriptive feedback to propel the learner forward. As Brookhart (2008) states, "good feedback contains information that a student can use in a timely and specific way." Feedback gives both teacher and learner their next steps.

Assessment of learning is gathered at or near the end of a period of learning. These strategically determined periods of learning may have focused on a concept, unit, or strand and could include rich tasks, demonstrations, projects, tests or exams *(Growing Success,* 2010*).*

Using a combination of observations, conversations and products gives us a rich set of data that we can use to assess and evaluate student learning. The Ministry of Education (2007) states: "a student's

achievement of the overall expectations is evaluated on the basis of his or her achievement of related specific expectations (including the process expectations)."

> *"The aim is to inform teacher or student judgments about the key decisions; 'Should I relearn…Practice again…Move forward…To What?'"*
>
> Hattie, 2012

3. How does inquiry-based learning support effective mathematics instruction and assessment?

Inquiry is a fluid and recursive approach to teaching and learning that is student-centred, and based on student needs. Students and teachers are co-learners through the inquiry process. An inquiry math classroom is one where students are actively wondering, posing questions, making connections to their own lives, planning ways to show their learning, and reflecting on what they have learned. In our math classrooms, students might be generating their own solutions to problems, discussing their solutions, and reflecting on their next steps.

An inquiry classroom supports effective assessment as teachers are constantly gathering data through observations and conversations, in an open and responsive environment that provides ongoing opportunities for feedback.

Click here to access the Monograph on Inquiry-based Learning.

> *"Inquiry-based learning is an approach to teaching and learning that places students' questions, ideas and observations at the centre of the learning experience. Educators play an active role throughout the process by establishing a culture where ideas are respectfully challenged, tested, redefined and viewed as improvable, moving children from a position of wondering to a position of enacted understanding and further questioning. (Scardamalia, 2002)"*
>
> Ministry of Education, 2013

4. How can we align assessment and instruction to support student learning?

Assessment and instruction to support student learning are aligned through the three-part lesson framework. The three-part lesson framework is a mindset to enable student thinking and learning through the use of *big ideas*. *Big ideas* are statements that link mathematical knowledge and skills into a coherent whole. The three-part lesson is a vehicle in which student thinking is made visible, and provides teachers multiple opportunities to assess student thinking (Fig. 1). A focus on the *big ideas* assists teachers with interpreting the curriculum as they plan differentiated lessons, based on a learning goal that is aligned to a *big idea*, and assessments to support the needs of all learners. *Big ideas* also help teachers and students to connect various mathematical concepts between strands and grades and make student thinking visible through the mathematical processes. Through the use of effective questions the teacher is able to draw out the key learnings of the lesson, to ensure students have a conceptual understanding.

When consolidating the student thinking, teachers may use high-yield communication strategies such as gallery walk, math congress and bansho. During this time, teachers check for conceptual understanding and that the learning goal has been met, and to inform their instructional next steps. Teaching for conceptual understanding through the use of rich tasks gives students the opportunity to reflect on their thinking, develop their thinking, and consolidate their thinking. Students have opportunity to consolidate and reflect on their learning by reflecting on similarities and differences between various student solutions, and to communicate their learnings as well as their struggles.

Click here to access the Monograph describing gallery walk, math congress and Bansho.

Fig.1 Assessment and Instruction Through the Three Part Lesson Framework

	Students are:	Teachers are:
When Activating Student Thinking	Engaging Organizing Questioning Reflecting Connecting	Inviting Encouraging Open Assessing Organizing Introducing Questioning
When Developing Student Thinking	Discussing Applying Reflecting Creating Listening Discovering Reasoning Questioning Exploring Connecting	Selecting strategies Listening Observing Assessing Scaffolding Questioning
When Consolidating Student Thinking	Discussing Sharing Listening Reflecting Questioning Connecting	Assessing Checking for conceptual understanding Pulling out the math Questioning

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Frequently Asked Questions

Attitudes to Math

"This curriculum…is based on the belief that all students can learn mathematics and deserve the opportunity to do so."

Ontario Curriculum for Mathematics, Grades 1-8

"There is a huge elephant standing in most math classrooms, it is the idea that only some students can do well in math."

Jo Boaler

1. What is a growth mind set?

A growth mindset is a belief that our skills and abilities can grow and develop over time with careful and patient practice and application. A fixed mindset is a belief that our abilities and intelligence are fixed traits that we are born with, and cannot be changed. Research done in recent years by Carol Dweck shows that, "students tend to have more of a fixed view of math skills than of other intellectual skills" ("Mindsets and Math/Science Achievement").

We believe that everyone can do math. The goal in our math classrooms is to help all our students shift from fixed to growth mindsets in mathematics. We want our students to stop saying, "I can't do math." Instead, we want all our learners to say, "I can't do it…yet", when confronted by challenges in mathematics.

"Students (and their teachers) can have different beliefs about intellectual abilities. Some believe that intellectual abilities are basically fixed—that people have different levels of ability and nothing can change that. In contrast, others believe that intellectual abilities can be cultivated and developed through application and instruction."

Dweck, (2008)

2. Why is a growth mindset important?

Dweck's research shows that students achieve more highly when they are given growth mindset messages. For example, when effort, struggle, and mistakes are praised, rather than ability, students are more likely to persevere with difficult work. Conversely, presenting the viewpoint that deeper mathematical thinking is accessible only to those who are born with it (so-called "math people"), gives students the opposite message. Students are more likely to give up if they feel that true math understanding is attainable for only a select few.

Careful and patient practice will lead to growth in mathematical skills and understanding over time. As Dweck notes, "even Einstein wasn't Einstein before he put in years of passionate, relentless effort." We also only hear about his successes, not his many struggles and mistakes which caused him great self-doubt and worry.

Growth mindsets in math can be built through careful and deliberate teacher messaging. Building a positive and open culture of math talk and collaboration, praising effort, not ability, and learning from mistakes are all actions that will help.

> *"Students' beliefs are correlated to their attitudes about, and achievement in, mathematics; if these dispositions are negative, learning is impeded and academic success is limited."*
>
> Colgan (2014)

3. What is math anxiety, and how do we help our students to overcome it?

We recognize that math anxiety is a very real thing for many of our students. Mathematical anxiety can be defined as "a feeling of tension, apprehension, or fear that interferes with math performance" (Ashcraft, 2002). The problem is compounded by the fact that many parents themselves experienced math anxiety in school, and math is portrayed as something to be loathed and feared in many popular culture and media sources. Children may hear from a young age that "math is hard", or "I can't do math", and internalize these messages themselves.

Students can become trapped in a very vicious cycle: they are anxious about math, and their anxiety interferes with their understanding and achievement, causing them to become even more anxious. A fixed mindset about their own ability becomes internalized, adding to their anxiety.

Three things we can do in our growth mindset math classrooms to reduce anxiety are: removing the emphasis on speed; focusing "on math making sense" (Small); and giving students open questions and rich tasks that allow for a variety of responses, not just "right" or "wrong". Boaler, Small and others have noted that an emphasis on speed promotes competition and anxiety. Those who are slower thinkers feel that they are not as smart as faster thinkers. In the past, math has often been reduced to a set of rules to be memorized, rather than a meaning making activity. Using more open questions and

interesting and rich tasks will allow students to focus on the growth in their own thinking, and not just on getting the right answer.

> *"The emphasis on "black and white" or "right and wrong" answers- no middle ground- is something that has contributed to math anxiety."*
>
> Small (2013)

4. How does having a growth mindset support me as a numeracy teacher?

First and foremost, as co-learners in math classrooms, we need to have growth mindsets ourselves. We believe that a collaborative math classroom should promote risk-taking, perseverance, and confidence, in both teachers and learners.

Teachers themselves should not be afraid to puzzle through math problems, activities, tasks, and investigations with our students. Being in front of a class of students causes our own anxiety, and fear of making mistakes. We need to give ourselves the same "breaks" we give our students, if it takes us time to find an answer, or if we're not sure of the answer right away (Small).

All teachers are numeracy teachers, just like all teachers are literacy teachers. Students will have the greatest belief in themselves as numerate learners if they see math connections across the whole curriculum. Schools should build communities of numeracy practice through a collaborative approach, beginning with conversations "about how we might support one another in working with students to develop their full potential in mathematics", and by exploring numeracy links across subjects and disciplines ("Supporting Numeracy" monograph, Ministry of Education). Helping our students make numeracy connections across the curriculum will make them more confident and persistent learners, and help them to demystify math.

> *"When we help our students make numeracy connections throughout the day, we are engaging them in understanding the many dimensions of our interconnected world."*
>
> Ministry of Education (2012)

5. Video clips can we use to build our understanding of math and mindsets.

Jo Boaler
A Growth Mindset in Math Class
Growth Mindsets
Develop a Growth Mindset
Timed Learning and Math Anxiety
Mindsets and Mistakes

Eduardo Briceno
The Power of Belief

Carol Dweck
Fixed vs. Growth Mindsets
A Study on Praise and Mindsets

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Frequently Asked Questions

Basic Facts and Operational Skills

"The Ontario Mathematics Curriculum is based on the belief that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving and are then guided carefully into an understanding of the mathematics principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum."

The Ontario Curriculum, Grades 1-8, Mathematics

"The required knowledge and skills include not only important mathematical facts and procedures but also the mathematical concepts students need to understand and the mathematical processes they must learn to apply."

The Ontario Curriculum, Grades 11 and 12, Mathematics

1. What are basic facts?

The Guides to Effective Instruction in Mathematics define basic facts as addition, subtraction, multiplication, and division using numbers from 0-9. Basic fact fluency is referred to as automaticity. Automaticity is the relatively effortless recall of facts and provides opportunities for students to engage more easily in the types of problem-solving we expect of them. Automaticity is a consequence of brief, frequent, interactive, engaging activities that provide students with repeated exposure to math facts.

Students' facility in using basic math facts often has a significant effect on their ability to view themselves as mathematicians and on their confidence. This confidence can be diminished if teachers pay excessive attention to memorization and speed and spend too little time helping students to understand the relationships and patterns in the basic facts. For many students, anxiety is associated with memorization tasks, particularly during speed drills and elimination games. The goal in math classrooms is to shift from simply memorizing facts to supporting increased understanding of math skills and concepts.

"*The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.*"

Boaler, 2015

2. What does it mean to master basic facts?

Memorizing basic facts is not the same as the mastery of basic facts. Mastering basic facts includes developing a conceptual understanding of the relationship between numbers and how these relationships can be extended into strategies for doing the computations in a meaningful, logical manner. Students who learn the basic facts using a variety of computational strategies will be able to extend these strategies and their understanding of number to multidigit computations. It is important to note key researchers (Jo Boaler, Marian Small, et al) have shown that an overreliance on memorized procedures prevents students from using mathematical reasoning. A combination of strategies which includes having students engage in problems where they need to use lots of basic facts to solve the problem, and participating in games where students are motivated to know facts fairly quickly, are recommended.

> *"I do not believe that it's a return to flash cards or 'mad minutes', whether on paper or digital, that we should be envisioning. I think we need the continued commitment to making calculations make sense..."*
>
> Small, 2011

3. What is mental math?

Mental computations, or mental math, are often used to calculate, estimate, or visualize using mathematical relationships and strategies that were previously learned conceptually, without the use of pencil/paper, calculators or thinking tools. Mental math strategies help to further develop students' basic fact fluency, thus leading to mastery. Mental math strategies include, but are not limited to, making 5's and 10's, making jumps of ten, and composing and decomposing numbers.

> *"Solving questions mentally helps to force a student to focus on relationships between numbers and the effect of number operations, as opposed to simply memorizing rules."*
>
> Small, 2008

4. What are operational skills?

A skill implies being able to do something well or proficiently. A student possessing operational skills in mathematics is able to competently perform basic operations with all types of numbers appropriate to their own development, as well as within a variety of algorithms. Basic fact fluency (i.e., automaticity) may assist students with the development of their operational skills, however, use of mental math strategies, thinking tools and technology would improve their proficiency with these skills while laying the groundwork for deeper understanding.

A student possessing operational fluency means a student knows *when* to perform the basic operations, however it does not necessarily indicate an ability to judge the reasonableness of the answers provided.

> *"…to master these bits and pieces [operational skills] is no more doing mathematics than playing scales on the piano is making music."*
>
> Van de Walle, 2007

5. Can we teach through problem solving without students knowing their basic facts?

When teaching through problem solving, the work of doing math begins with problems which contain important and interesting math, and lead to the development of procedural and conceptual understanding. It is an opportunity for students to make sense of mathematical concepts. Teaching through problem solving gives students the opportunities to make sense of their thinking and the thinking of others through the mathematical processes. Having basic fact fluency (i.e. automaticity) frees the brain to focus on the work of doing math and the mathematical concepts.

> *"The highest achievers in the world are those who focus on big ideas in mathematics, and connections between ideas. Students develop a connected view of mathematics when they work on mathematics conceptually and blind memorization is replaced by sense making."*
>
> Boaler, 2015

6. How can technology support the development of basic facts and operational skills?

Technology plays an essential role in enhancing the learning and doing of mathematics. They are important problem solving tools which help support reasoning, promote thinking and conceptual development, and can also enhance operational fluency. When purposefully used, computer software, web-based applications, apps, and calculators can help students practice skills, reinforce what they have learned, and build confidence.

Calculators in particular have sparked many debates regarding the role of technology in the mathematics classroom. A meta-analysis by Smith (1997) on the role of calculators showed that their use improved conceptual knowledge, problem solving skills, and computation, and did not hinder pencil-and-paper skills. Furthermore, it is the recommendation of the National Council of Teachers of Mathematics that calculators be used in all aspects of math instruction, from the development of concepts to the acquisition of operational skills. Calculator use should only be restricted when students are focusing on mental math strategies.

As with all forms of technology in the mathematics classroom, teachers and students must consider when it is purposeful to use them.

> *"Although students must develop basic operational skills, calculators and computers can help them extend their capacity to investigate and analyse mathematical concepts and reduce the time they might otherwise spend on purely mechanical activities."*
>
> Ministry of Education, 2005

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COMPREHENSIVE NUMERACY STRATEGY K-12

2014 – 2017 – Updated August 2015

Numeracy is our curricular focus in the Peel District School Board. We have explicitly defined our strategy to empower all educators to support the implementation of improved numeracy instruction in Peel classrooms. Our Comprehensive Numeracy Strategy has 3 drivers: the instructional focus, instructional strategies and assessment.

"[The Ontario Mathematics Curriculum] is based on the belief that students learning mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem-solving and are then guided carefully into an understanding of the mathematical principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum." (The Ontario Curriculum, Grades 1-8)

"Today's mathematics curriculum must prepare students for their future roles in society. It must equip them with an understanding of important mathematical ideas; essential mathematical knowledge and skills; skills of reasoning, problem-solving and communication; and , most importantly, the ability and the incentive to continue learning on their own." (The Ontario Curriculum, Grades 11 and 12)

Our Instructional Focus	Educators across all grades and subject areas will focus on teaching and learning through the lens of numeracy with an emphasis on proportional and spatial reasoning in order to make student learning visible.
Instructional Strategies	Educators will use learning goals and co-created success criteria, teach using the 3 part lesson framework, employ rich tasks and use thinking tools and manipulatives in order to support student learning and thinking.
Assessment for Learning	Educators will use descriptive feedback and formative assessment in order to improve student achievement.

As we move along our learning journey focusing on effective instruction and assessment practices to improve student achievement in mathematics, the key areas of focus for the Comprehensive Numeracy Strategy for 2015-2016 are:

- Deepening our mathematical content knowledge for teaching
- Use of rich tasks for instructional and assessment purposes
- Numeracy and how it relates to Mathematics
- Assessment for Learning to enhance instruction and student learning

COMPREHENSIVE NUMERACY STRATEGY K-12

2014 – 2017 – Updated August 2015

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
The Numeracy Team develop core professional learning opportunities on the 3 drivers differentiated for Superintendents, Principals, Instructional Coaches, and educators.	• Numeracy Team comprised of math educators from Peel and from the province, including, but not limited to Mathematical Literacy, Coordinator, Literacy Coordinator, Assessment Coordinator, Coordinating Principals School Effective Leads, Selected Superintendents of Education, Associate Director, ETFO and OSSTF Representatives	• CISESS Superintendents • CISS Superintendent of Education • Mathematical Literacy Instructional Coordinator	• Continuing September 2015 • 3 meeting per year (November, February and May)
Numeracy Strategy addressed at Starting Point. Key messages will be reviewed. Successes and next steps will be outlined developed, A focus will be placed on: • Showcasing how math is everywhere • We are ALL teachers of Numeracy - This is 21st century teaching and learning • 3 Core Drivers remain unchanged • Success from the first year • Key areas of focus for 2015-2106	• Starting Point speech	• Director • Communications Support Services • Numeracy Team	• September 2015
All schools will have a numeracy goal and theory of action as their main school success focus	• School Success Planning module to be revised to facilitate numeracy goal setting	• Superintendents of Education • Principals • School teams	• By November 15, 2015

Principals work with staff teams to promote a school culture that supports numeracy cross the curriculum based on ***Balanced Mathematics Instruction K-12*** document	• *Balanced Mathematics Instruction K-12* document • Workshops: Facilitation Skills for Department Heads • *EngageMath* website • Mathematics Learning in a Play-Based Environment document	• Superintendents of Education • Principals • Secondary Department Heads	• Continuing September 2015 and ongoing
All Principals will focus their professional learning opportunities at the school level (staff meetings, release days, department meetings, PL Days) on their numeracy goal Provide meaningful, focused support and professional development for lead teachers and secondary Department Heads	• *EngageMath* website	• Principals • Instructional Coaches (to build capacity with school teams) • CISS Math Team (to include Mathematics Resource Teachers and School Effectiveness Leads)	• Continuing October – June 2016
Principals and/or VPs will attend portions of *all* collaborative inquiries (K-6 networks, 7-10 networks, school sessions) related to numeracy with their teaching teams Key areas of focus: • Deepening our mathematical content knowledge for teaching • Use of rich tasks for instructional and assessment purposes • Numeracy and how it relates to Mathematics • Assessment for Learning to enhance instruction and student learning	• Resources developed by CISESS staff & Math Team in order to support building a unit of study, assessment & reporting	• Principals, Vice-Principals • Teachers • Program Coordinators • Instructional Coaches • CISS Math Team	• Continuing October 2015 – June 2016

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
All schools K – 12 will participate in numeracy focused collaborative inquiry networks. 7 -10 Networks will continue to support cross-panel family of schools groupings. K-6 networks will again focus on grades 3 to 6 with a school choice of participation in centrally supported and directed networks or in self-selected, self-directed networks. Selected schools will have the opportunity to use the resource 'Dreambox' as a pilot project to support their professional learning and their work with students. Early Years Lead Network Inquiries (K-2) will focus mathematical learning in the primary grades. Identified schools in the SSI initiative will adopt numeracy as the focus of their collaborative work – specifically supporting teachers of grade 9 and 10 applied mathematics.	• Math Team • Engage Math website • Curriculum documents • Ministry resources • Dreambox on-line software • Peel resources	• Mathematical Literacy Coordinator • SEL's • Math Team • Dreambox Learning • Early Years Coordinator	• October 2015 to February 2016
The Comprehensive Numeracy Strategy K-12 is used as the main focus for the submission of the Board Improvement Plan Schools develop School Success Plans based on Comprehensive Numeracy Strategy	• Board Improvement Plan integrally aligned with Comprehensive Numeracy Strategy • Reviewed with ISS Team in August	• CISESS Superintendents • CISS Math Team	• By Fall 2015
The CISS Math Team will provide in-depth, differentiated professional learning for **instructional**	• Comprehensive Numeracy Strategy K-12	• CISS Math Team	• First week of September 2014

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
coaches on the 3 drivers referenced in the Comprehensive Numeracy Strategy K-12. Professional learning will include PRIME training to support the instruction coaches' content knowledge for teaching mathematics	• Balanced Mathematics Instruction K-12 • *EngageMath* Website	• Nelson (PRIME) Representatives	(will need to be revised) • Friday meetings • September 2015 – December 2015

COMPREHENSIVE NUMERACY STRATEGY K-12

2014 – 2017 – Updated August 2015

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES

The CISS Math Team will provide regular professional learning for **the ISS Team** on the curriculum	• Comprehensive Numeracy Strategy K-12
	• Balanced Mathematics Instruction K-12
	• Professional Resource: *Making Math Meaningful*
	• *EngageMath* Website
	• Curriculum Documents
		• CISS Math Team	• October 2015 – February 2016 at:
			  • Pre Board meetings
			  • ISS Meetings

Educators will be encouraged to engage in after-hours professional learning related to the teaching of Mathematics	• Ministry AQ courses • Dinner & Dialogue series	• Mathematical Literacy Coordinator • Superintendents of Education	• Continuing September 2015
At SOE meetings, Superintendents of Education will integrate and align the language of Instructional Rounds and the Comprehensive Numeracy Strategy K-12	• Comprehensive Numeracy Strategy K-12 • Balanced Mathematics Instruction K-12	• Superintendents of Education	• Continuing September 2015 and ongoing
Provide explicit links to recommended Transformational Practices strategies to be used to implement Comprehensive Numeracy Strategy	• Mathematical Literacy Coordinator • CISS Math Team • Instructional Coaches	• Mathematical Literacy Coordinator • Assessment Coordinator • Instructional Coaches	• continuing September 2015 and ongoing
Explore and identify a common assessment for proportional and spatial reasoning K-12	• SEF Framework • Comprehensive Numeracy Strategy K-12 • Balanced Mathematics Instruction K-12	• Mathematical Literacy Coordinator • Assessment Coordinator	• September 2015 to June 2016

COMPREHENSIVE NUMERACY STRATEGY K-12

2014 – 2017 – Updated August 2015

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
Develop a data monitoring strategy that includes indicators of student achievement in the six areas of the **Balanced Mathematics Instruction K-12** Develop clear established success criteria which link to the 3 core drivers	• Comprehensive Numeracy Strategy K-12 • Balanced Mathematics Instruction K-12	• Mathematical Literacy Coordinator • Math Team • Secondary Department Heads	• By February 2016 and ongoing
CISESS Celebrating Learning (Literacy/Numeracy Conference)	• Guest speakers to include Cathy Marks Krpan • Literacy/Numeracy Break-out sessions	• Mathematical Literacy Coordinator • Literacy Coordinator	• November 18 &19, 2015 • Pearson Convention Centre

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
Parent Numeracy Conference	• Keynote speaker • Parent Workshops • Vendor display	• Communications Support Services • All CISESS Coordinators	• Spring 2016
Engage parents in the Comprehensive Numeracy Strategy through various strategies (eg. Math Nights for families, ,Welcome to Kindergarten Events, PRO Grants, and camera-ready articles) that explain the broad strategy and deal with the six areas of the **Balanced Mathematics Instruction, K-12** document Continue to produce age appropriate articles for newsletters on the six components of the *Balanced Mathematics Instruction, K-12* document: • Basic Facts and Operational Skills • Attitudes to Math • Approaches to Instruction/Assessment • Teaching Through Problem-Solving	• Address the successes from the previous year, key areas of focus for 2015-2016, brief outline of the Comprehensive Numeracy Strategy, and address the 3 drivers in the form of a camera-ready article • Minimum of 6 camera-ready articles for inclusion in school, one for each column of the	• Communications Support Services • Camera-ready articles to support presentations at School Council meetings • CISS Math Team	• Continuing September 2015 and ongoing

COMPREHENSIVE NUMERACY STRATEGY K-12

2014 – 2017 – Updated August 2015

ACTION ITEM	RESOURCES	WHO IS RESPONSIBLE?	TIMELINES
• Purposeful Practice • Purposeful Use of Resources	*Balanced Mathematics Instruction* document • Section of each school's website to include camera-ready articles, *Balanced Mathematics Instruction* document, etc. based on grade levels in school		
Review and gather feedback of the Comprehensive Numeracy Strategy – Year 2	• Transformational Practices document • Comprehensive Numeracy Strategy K-12 • Balanced Mathematics Instruction K-12	• Mathematical Literacy Coordinator • Assessment Coordinator • Numeracy Team • CISESS Superintendents • Superintendents of Ed.	By May 2016
Revise and publish Comprehensive Numeracy Strategy for Year 3	• Numeracy Team	• Numeracy Team • CISESS Superintendents • Superintendents of Ed. • Mathematics Literacy Coordinator	June 2016

peel District School Board

"[The Ontario Mathematics Curriculum] is based on the belief that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving and are then guided carefully into an understanding of the mathematical principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum."

The Ontario Curriculum, Grades 1-8, Mathematics, p.4

Balanced Mathematics Instruction, K-12

"Today's mathematics curriculum must prepare students for their future roles in society. It must equip them with an understanding of important mathematical ideas; essential mathematical knowledge and skills; skills of reasoning, problem solving, and communication; and, most importantly, the ability and the incentive to continue learning on their own."

The Ontario Curriculum, Grades 11 and 12, Mathematics, p.4

Attitudes to Math

A growing body of research (e.g., Dweck, Boaler, et. al.) demonstrates that the beliefs that educators and students have about math is critical to student achievement in math.

We believe that all students can learn math. We believe that all teachers are numeracy teachers in the same way that all teachers are literacy teachers. We believe that effective math instruction promotes risk-taking, perseverance, and confidence. We believe that effective math instruction makes connections across disciplines. We believe that math instruction must support all pathways.

"Students who believe that intelligence or math and science ability is simply a fixed trait (a fixed mindset) are at a significant disadvantage compared to students who believe that their abilities can be developed (a growth mindset)." (Carol Dweck, "Mindsets and Math/Science Achievement")

Approaches to Instruction/ Assessment

Teachers design math instruction/assessment to support learners in achieving the expectations of The Ontario Curriculum.

A comprehensive mathematics program includes a variety of instructional/assessment approaches in rich learning contexts, and focuses on the development of conceptual and procedural understanding, skill development and problem-solving. A balanced program begins with the learner profile and includes guided/direct instruction, as well as opportunities for student inquiry in which students generate their own solutions. A variety of groupings for collaborative learning with peers as well as time for independent learning are essential.

Key Transformational Practices (e.g., 3-Part Lesson) are used in all math classrooms.

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways." (The Ontario Curriculum Grades 9 and 10, Mathematics, p.23)

Teaching Through Problem Solving

Teaching through problem solving is not the same as solving word problems. When students engage in problematic situations, they become curious; motivated to explore and generalize mathematical ideas. Problems are carefully selected and differentiated to be accessible yet challenging for all students.

"[Problem solving] can be used as the means of introducing concepts rather than simply engaging students in applying or practicing mathematical procedures." (The Report of the Expert Panel on Mathematics in Grades 4-6, p.11)

A problem solving approach also develops what Fullan characterizes as a vitally important 21st century teaching and learning skill. Students must engage in "Critical thinking and problem solving... [to] think critically to design and manage projects, solve problems, make effective decisions using a variety of digital tools and resources." (Great to Excellent, p. 9)

Purposeful Practice

A balanced math program provides opportunities for students to practice and consolidate skills and procedures.

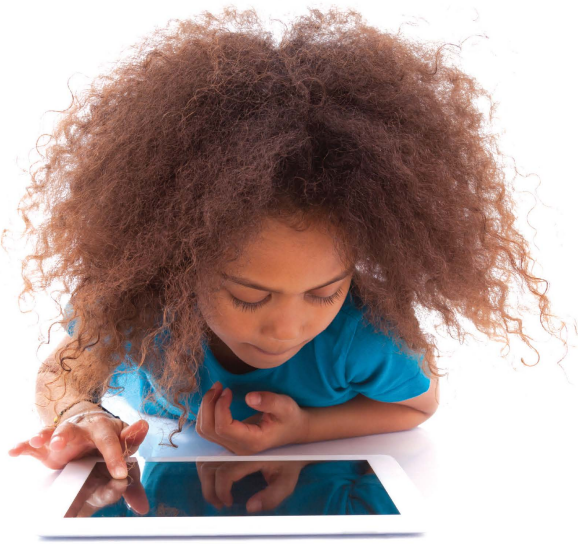
Purposeful practice strengthens the connection between skills, concepts, strategies and thinking. It improves speed and accuracy and helps students remember concepts, facts and procedures.

Once conceptual understanding has been established, purposeful practice helps students develop computational and procedural fluency.

"Children need time to practise and consolidate skills, balanced with time to put those skills to use in a problem solving context." (The Report of the Expert Panel on Early Math in Ontario, p.31)

The important thing is that practice is purposeful and responsive to each individual student's needs.

A balanced math program uses games effectively to facilitate purposeful, responsive, and individualized practice.



Basic Facts and Operational Skills

Operational skills and basic facts are important because they support efficiency and the ability of students to judge the "reasonableness" of a solution in mathematics. Therefore, students are expected to master basic facts and operational skills. The Transformational Practice "Teaching Basic Facts and Operational Skills" is a key starting point for planning instruction/assessment.

Operational skill is more than fast recall on math drills. It involves understanding why a fact or procedure makes sense and how it connects to other concepts and skills. Students use mental math to calculate, estimate, and visualize math concepts and strategies. When students develop both fluency and understanding, skills become tools to help them solve problems.

"Skills and understanding are especially critical when tackling challenging problems." (C. Sealy, Balance is Basic in Faster Isn't Smarter, 2009, p.2)

Purposeful Use of Resources

A variety of resources and tools are used to engage students and support learning. Resources and tools are selected thoughtfully, informed by curricular expectations and responsive to student learning needs.

Manipulatives (i.e., thinking tools), technology, media, textbooks, and professional materials (e.g. Guides to Effective Instruction, TIPS, Edugains.ca) are all valuable resources. Using a resource purposefully is different than following the resource lesson by lesson.

"Manipulatives...are central to effective instruction and have the capacity to greatly improve and deepen student understanding." (The Report of the Expert Panel on Mathematics in Grades 4-6, p. 25)

The important thing is that any resource or tool is used purposefully to respond to the needs of the learner.

A comprehensive math program balances skills, concepts, strategies, and thinking.

Frequently Asked Questions

Purposeful Practice

"The teacher's ability to deliver effective mathematics instruction is the most powerful factor in determining how well students learn mathematics. Effective mathematics instruction is enhanced when teachers develop and deepen their own understanding of mathematics, of student learning, and of strategies that promote mathematical proficiency. This understanding can help to ensure that teachers are informed and critical thinkers who are able to make wise choices about activities, strategies, and resources and who are able to provide a comprehensive program that supports children's development of mathematical proficiency".

Guide to Effective Mathematics Instruction, Volume 1. 2006. p. ix.

1. What is purposeful practice?

Purposeful practice is practice that is thoughtful and deliberate while being responsive to individual students' needs to enhance students' conceptual and procedural understanding of mathematics. Comprehensive mathematics instruction begins with assessment for learning to determine students' strengths and needs, and informs the level of support required within the gradual release of responsibility. In this responsive practice, the instructional decisions are based on descriptive evidence of learning that guides the teacher in differentiating instruction. Through purposeful practice there is a greater likelihood for all students to understand and make sense of mathematical concepts and ideas.

"Teachers are designers. An essential act of our profession is the design of curriculum and learning experiences to meet specified purposes. We are also designers of assessments to diagnose student needs to guide our teaching."

Wiggins, G. & McTighe, J., 1998

2. How does purposeful practice support students' mathematical thinking?

Mathematical thinking is central to the learning and teaching of mathematics. If student learning and understanding is the intended outcome, and we believe that learning is a product of thinking, then we need to purposefully align our instruction and assessment practices to support student thinking (Ritchhart, Church, & Morrison, 2011). Students' mathematical thinking is developed and made visible through the three-part framework. This framework incorporates the use of rich tasks with clearly established success criteria and learning goals. It is through this framework that student thinking is made visible and teachers are able to differentiate their instruction and assessment as they respond to and adapt to individual students' needs. Purposeful practice enables teachers to further develop student thinking by differentiating what each student practices as they make sense of mathematical concepts and ideas. It is only when students make their thinking visible, through purposeful practice, that we can properly assess their understanding, provide descriptive feedback, and improve student learning.

> *"For thinking to occur students must first have something to think about and be asked to think. We as teachers must create opportunities for thinking."*
>
> *Ritchhart, R., Church, M., & Morrison, 2011*

3. How can we support student practice within the consolidation phase of the three-part lesson framework?

During the consolidation phase of the three-part lesson framework students reflect on their thinking and the thinking of others to enhance their understanding of mathematical ideas. They are also given the opportunity to practice applying new ideas and strategies. Part of a comprehensive mathematics program involves planning to ensure that students receive sufficient, multiple and varied opportunities for practice. The outcomes of practice that support mathematical thinking may include, but are not limited to, visualization, metacognition, development of alternative and flexible strategies. (Small, 2013). Purposeful practice, within the consolidation phase, supports the development of students' mathematical thinking and has the potential to increase students' mathematical understanding, creativity and confidence in the way they think about themselves as mathematicians.

> *"Children need time to practise and consolidate skills, balanced with time to put those skills to use in a problem solving context."*
>
> *The Report of the Expert Panel on Early Math in Ontario, 2003*

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Frequently Asked Questions

Purposeful Use of Resources

To sustain the effectiveness of a differentiated instructional approach, it is critical to conduct ongoing, authentic assessment, and then to adjust strategies and resources according to the assessment results.

Learning for All: A Guide to Effective Assessment and Instruction for All Students, Kindergarten to Grade 12, p. 19

1. How does the purposeful use of resources support the learning and teaching of mathematics?

Student inquiry, teacher inquiry, skills of the 21st century, content knowledge for teaching, instruction and assessment are just a few of the many elements that constitute effective teaching practices in the mathematics classroom. There are a myriad of resources to support the many components of the learning and teaching process, driven by student thinking.

Resources are selected to inform instruction and assessment based on the learning needs of the student determined through purposeful practice. Resources focusing on deepening teachers' pedagogical knowledge and content knowledge for teaching can be used to support teaching through problem solving to make student thinking visible.

"As teachers become more familiar with which ideas are more complex for students and why, they are better able to ensure that their instruction is at the appropriate developmental level for students, and that it challenges students' mathematical conceptions in appropriate ways."

Making Math Meaningful to Canadian Students, K–8, p.12

2. How does the purposeful use of resources support students' mathematical thinking?

A variety of resources and tools can be use to engage students and support students' learning and mathematical thinking. Resources (e.g., Gap Closing, TIPS4RM, Guides to Effective Instruction, Continuum and Connections, manipulatives-thinking tools) are used to activate, develop and consolidate student thinking. Learning and teaching in the context of the 21st century focuses on developing the skills of communication, creativity, collaboration and critical thinking; manipulatives, technology, and media are purposefully used in the development of those skills.

Using resources purposefully to deepen students' conceptual and procedural understanding of mathematics is part of being responsive to students' individual learning needs. It does not imply adhering to a prescribed series of lessons that do not take these needs into consideration.

> *Differentiating instruction based on student readiness involves knowing where particular students are on the learning continuum, then planning program features and instructional strategies, resources, and supports to meet them where they are and move them along this continuum.*
>
> *Learning for All: A Guide to Effective Assessment and Instruction for All Students, Kindergarten to Grade 12, p. 18*

3. How does the purposeful use of resources support teacher inquiry?

The role of teacher inquiry is a critical component of the daily work of teachers. Ministry of Education (2010) states, "inquiry positions the teachers as an informed practitioner refining planning, instruction and assessment approaches in the continual pursuit of greater precision, personalization and innovation. A focus on student learning drives inquiry. Data generated from student actions and work compels teachers to investigate new, engaging and relevant questions about how and what their students learn". The purposeful use of resources within the teacher inquiry process deepens teachers' professional knowledge, both pedagogical and content knowledge for teaching, and informs practice.

4. How do thinking tools support student inquiry and enhance students' mathematical thinking?

An inquiry classroom is one where students are actively wondering, posing questions, making connections to their personal experiences, planning ways to show their thinking, and reflecting on their next steps. Students are actively constructing knowledge by exploring, investigating, discovering, and creating. Students are encouraged to use thinking tools, such as manipulatives, to enhance the learning process as they engage in rich mathematics tasks. Manipulatives – thinking tools – are physical or virtual objects that can be used to demonstrate or model mathematical thinking. A conceptual understanding of mathematical concepts is brought about through purposeful teaching with the support of thinking tools.

> *"Manipulatives…are central to effective instruction and have the capacity to greatly improve and deepen student understanding."*
>
> *The Report of the Expert Panel on Mathematics in Grades 4-6, p.25*

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